**Share ride**

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2019 Fall

ABSTRACT

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# NOMENCLATURE

|  |  |  |
| --- | --- | --- |
| Parameter | Definition | Unit |
|  | The fleet of vehicles |  |
|  | Number of vehicles |  |
|  | Vehicle capacity |  |
|  | The set of requests |  |
|  | Number of requests |  |
|  | Request |  |
|  | Cost function |  |
|  | Constraints |  |
|  | Passengers for vehicle |  |
|  | Waiting time | min |
|  | Pickup time |  |
|  | Request time |  |
|  | Maximum waiting time |  |
|  | Total travel delay | min |
|  | Drop-off time |  |
|  | Earliest possible time |  |
|  | Maximum travel delay |  |
|  | Origin |  |
|  | Destination |  |
|  | Maximum number of passengers |  |
|  | Assignment |  |
|  | Assigned requests |  |
|  | Unassigned requests |  |
|  | A trip, set of requests combined and served by a single vehicle |  |

# 1 DESIGN PROBLEM STATEMENT

Taxi services are a vital part of urban transportation, and a considerable contributor to traffic congestion and air pollution causing substantial adverse effects on human health. Sharing taxi trips is a possible way of reducing the negative impact of taxi services on cities. The share ride is serving multiple rides with a single trip.

The problem with the share ride is the expense of passenger discomfort quantifiable in terms of a longer travel time. This study focuses on the optimization analysis of routing strategy to minimize the overall delay time due to waiting and sharing while meeting all requests.

# 2 MATHEMATICAL MODEL

## 2.1 Variables and objectives

Consider a fleet of vehicles of capacity , the maximum number of passengers each vehicle can have at any given time. The problem is to optimally assign online travel requests to vehicles and find optimal routes for the vehicle fleet. Each travel request consists of the time of request, a pickup location and a drop-off location.

The task is for batch assignment of a set of requests to a set of vehicles , which minimizes a cost function , satisfies a set of constraints and allows for multiple passengers per vehicle. A request is defined as a vector, indicating: origin, destination, time of request , latest possible pickup time (initially given as the maximum wait time after the request time), pickup time , drop off time , and shortest possible time the destination could be met , which is defined as , the request time and the time to travel between origin and destination without fulfilling other requests. The set of passengers is denoted as for vehicle .

A Trip, is the set of requests that can be combined and served by a single vehicle. One of these trips may have one or more candidate vehicles for executing the trip completely. The cost is defined as the sum of delays over all assigned requests and passengers, plus a large constant for each unassigned request. The total travel delay is , where is the drop-off time and is the earliest possible time at which the destination could be reached if the shortest path between the origin and the destination was followed without any waiting time. The total travel delay includes both the in-vehicle delay and the waiting time. Given an assignment of requests to vehicles, denote the set of requests that have been assigned to some vehicle by , and the set of unassigned requests by due to the constraints and fleet size.

The objective function is

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

## 2.2 Constraints

The following constraints are considered:

1. For each request , the waiting time , given by the difference between the pickup time and the request time , must be below a maximum waiting time , for example, 2 min, i.e.

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

1. For each passenger or request the total travel delay must be below a maximum travel delay , for example, 4 min, i.e.

|  |  |  |
| --- | --- | --- |
|  | . | (3) |

1. For each vehicle , a maximum number of passengers is considered, for example, capacity 4, i.e.

|  |  |  |
| --- | --- | --- |
|  | . | (4) |

1. Each vehicle is assigned to one trip at most.
2. Each request is assigned to a single vehicle or ignored.

## 2.3 Assumptions

The following assumptions are made:

1. The vehicles and the requests of passengers are predetermined. That is, during the analysis, the number of vehicles and requests, and the time sequence of requests are kept constant.
2. The capacity of each vehicle is assumed to be the same, for example 4.
3. The driver of a vehicle does not reject any assigned request and accept immediately. That is, the driver’s reaction time for an assigned request is ignored.

# 3 MODEL ANALYSIS

The constrained optimization problem is solved via four steps (Figure 1), which are:

1. computing a pairwise request-vehicle shareability graph (RV-graph) (Figure 1 *B*);
2. computing a graph of feasible trips and the vehicles that can serve them (RTV-graph) (Figure 1 *C*);
3. solving an integer linear program (ILP) to compute the best assignment of vehicles to trips (Figure 1 *D*); and
4. rebalancing the remaining idle vehicles (Figure 1 *E*).

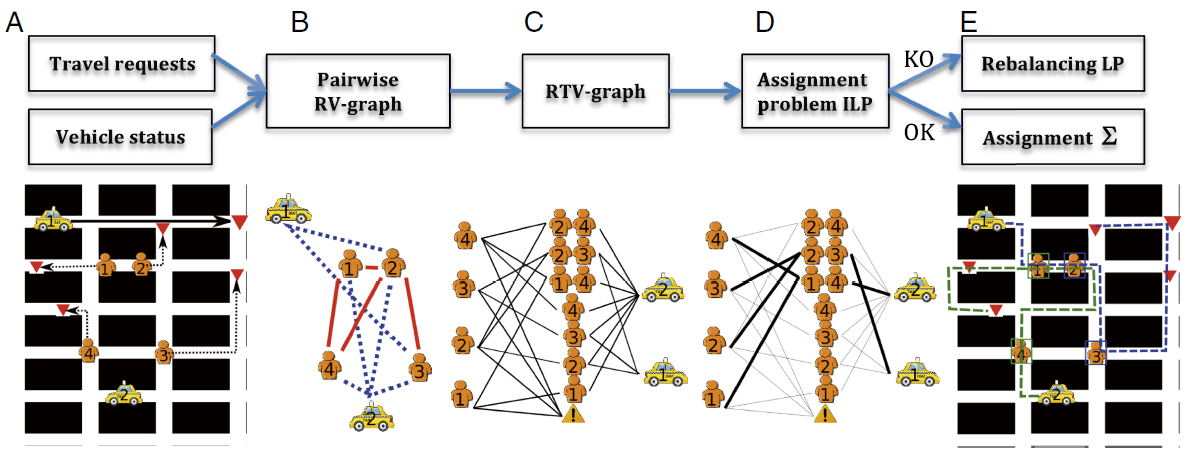


Figure 1 Schematic overview of the proposed method for batch assignment of multiple requests to multiple vehicles.

The problem of designing a framework for multi-vehicle multi-request routing must address the following fundamental problems:

* Computing an optimal assignment of a set of requests to a set of vehicles (given capacity of vehicles) (This is the main problem)
* Allowing for continuous operation and assignment of incoming requests to vehicles which are in the fleet (set of vehicles)
* Enabling rebalancing of fleet

The first problem is solved by optimizing the cost objective, i.e. Eq. (1) subject to the constraints in section 2.2. The problem, given the constraints, is NP hard, however, and thus a near optimal solution may suffice. In the Ref. [1], the sub-optimal solution is considered sufficient for practical applications in real time, and the sub optimality of the problem is claimed to “[possibly] incrementally improve over time.”

The second problem, anytime property and continuous assignment, are omitted from this portion of the report.

The third problem, fleet rebalancing, is omitted from our problem planning in this preliminary report as well.

# 4 OPTIMIZATION STUDY

## Overview

## 4.2 Pairwise graph of vehicles and requests (RV-graph)

## 4.3 Graph of candidate trips and pick-ups (RTV-graph)

## 4.4 Optimal assignment of trips to vehicles

In the following, the optimization method to optimally assign trips to vehicles is described.

### 4.4.1 Variables

A binary variable is introduced for each edge between a trip and a vehicle in the RTV-graph. If then vehicle is assigned to trip . Denote by the set of indexes for which an edge exists in the RTV-graph.

An additional binary variable is introduced for each request . These variables are active , if the associated request cannot be served by any vehicle and is ignored.

Denote the set of variables

### 4.4.2 Cost

The cost function, equivalent to in Equation (1), is given by

where the individual costs are given by the sum of delays,

and is a large enough constant to penalize ignored requests.

### 4.4.3 Constraints

Constraint 1: Each vehicle is assigned to a single trip at most,

where denotes the indexes for which an edge exists in the RTV-graph.

Constraint 2: Each request is assigned to a vehicle or ignored,

where denotes the indexes for which an edge exists in the RTV-graph and denotes the indexes for which an edge exists in the RTV-graph. This is, the trips of which the request forms part and the vehicles that can service each of those trips.

### 4.4.4 Assignment

The optimal assignment is found by solving a mixed-integer linear programming (MILP) optimization defined by the aforementioned variables, cost and constraints.

The MILP is done using function in Matlab. The code can be seen in Appendix. The syntax of intlinprog is

where is the objective linear function, is the number of integer variables, and define the inequality constraints, and define the equality constraints, and are lower and upper bond respectively.

For this problem, x is the combined solutions of and . Denote the number of requests, trips and vehicles are , and , respectively. The number of all ’s is , and the number of all ’s is . Thus, the dimension of is . In this study the order of elements in x is as follows: for vehicle 1, …, for vehicle , for request 1, …, for request .

# 5 DISCUSSION OF RESULTS

## 5.1 XXXX

# REFERENCES

[1] J. Alonso-Mora, S. Samaranayake, A. Wallar, E. Frazzoli, and D. Rus, "On-demand high-capacity ride-sharing via dynamic trip-vehicle assignment," *Proceedings of the National Academy of Sciences,* vol. 114, no. 3, pp. 462-467, 2017.